The Mass and Width of the Z^{0} Resonance

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The mass and width of the Z^0 resonance

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Precise values for the mass and total and partial decay widths of the Z^0 have been obtained by the four major experiments at LEP from the analysis of data collected during 1989 and 1990. The results are reviewed and are shown to be in excellent agreement with the predictions of the Standard Model of particle physics.

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1. Introduction

The first year of operation of the Large Electron Positron storage ring (LEP) at CERN has already allowed a new level of precision to be reached in experimental investigations of the Standard Model of particle physics. Data have been taken at a series of e^+e^- centre of mass energies between 88 GeV and 95 GeV that span the mass range of the Z^0, one of the gauge bosons of the electroweak interaction. Measurement of the resonant lineshape corresponding to the formation and decay of the Z^0 leads to the determination of its mass, a fundamental constant of electroweak theory. The knowledge of cross sections for the production of hadrons and of charged lepton–antilepton pairs allows the partial decay widths of the Z^0 to quarks and leptons to be determined and the predictions of electroweak theory to be tested with great sensitivity. The Z^0 invisible decay width comes from light matter that interacts only through the weak force and hence can be used to determine the number of species of light neutrino that couple to the Z^0.

This review summarizes the present results from the four major LEP experiments (ALEPH, DELPHI, L3 and OPAL) on the Z^0 resonance parameters. These come from the preliminary analyses of all data taken to date, totalling more than 625000 Z^0 decays. The results are compared with the predictions of the Standard Model.

2. Z^0 lineshape measurements

(*a*) *The experimental cross section*

The four experiments seek to identify and distinguish electron–positron interactions producing fermion–antifermion pairs in the final state, $e^+e^- \rightarrow f\bar{f}$. Here the fermion f is either a quark (q) or one of the charged leptons, the electron (e), muon (μ) or tau (τ). Quarks and antiquarks produce strongly interacting particles, hadrons, within the detector.

The cross section, σ, for the production of a given final state is calculated from the relationship

$$\sigma = (N_S - N_B)/\epsilon \int \mathscr{L}\,dt,$$

where N_S is the number of events selected, including both signal and background, N_B is the number of background events and ϵ is the product of the experimental

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Table 1. *Numbers of events, systematic errors and integrated luminosities of the hadronic event selections and the systematic errors on the luminosity determination*

experiment	number of events	systematic error (%)	integrated luminosity	luminosity systematic error (%)	
			pb^{-1}	experiment	theory
ALEPH	166000	0.2	—	0.6	0.5
DELPHI	119476	0.5	5.68	0.8	1.0
L3	115097	0.4	5.44	0.75	0.7
OPAL	165694	0.35 expt 0.5 theory	7.78	0.7	0.5

acceptance and efficiency. The integrated luminosity for the measurement, $\int \mathscr{L}\,\mathrm{d}t$, is the number of events expected per unit cross section, and is determined by the profile, overlap functions and integrated intensity of the electron–positron beams.

Throughout the first year of data-taking and analysis there have been notable successes in reducing both theoretical and experimental systematic errors associated with the measurements of N_s, N_B, ϵ and $\int \mathscr{L}\,\mathrm{d}t$, allowing full use to be made of the increasing statistical precision of the data. Details of the techniques used can be found in the experimental references (Decamp *et al.* 1990; Abreu *et al.* 1990; Adeva *et al.* 1990; Akrawy *et al.* 1990*a*).

For the measurement of N_s, each detector was designed and built with the capability of distinguishing between hadronic and the different leptonic final states. The features used are the tracking of charged particles in a magnetic field, the measurement of the energy and momentum of particles and the ability to identify electrons and muons within the detector. The degree of emphasis placed on the different components of information and the details of the experimental cuts used vary from one experiment to another and are to some extent complementary, being matched to the particular strengths of the individual detector. The number of background events within the selection, N_B, is estimated both from the data themselves and from Monte Carlo simulations of the detector response to known physics processes. The experimental acceptance and efficiency are similarly determined.

The numbers of events, acceptance ranges and systematic errors in the event selection are given in tables 1 and 2, corresponding to the preliminary analysis of the complete data collected between August 1989 and August 1990.

In an additional analysis, ALEPH and DELPHI identify lepton pair events irrespective of the flavour of the lepton. This removes some correlated systematic errors caused by lepton misidentification between the individual channels and so provides good data for use if lepton universality is assumed.

The integrated luminosity, $\int \mathscr{L}\,\mathrm{d}t$, is separately determined by each experiment from measurements of the number of Bhabha events scattered through small angles (less than 120 mrad), using purpose-built forward detectors. The techniques are detailed in the experimental references. At these small angles Bhabha scattering is dominated by QED processes, and so is insensitive to Z^0 effects (Z^0 exchange and γ–Z^0 interference). The luminosity determination requires the theoretical calculation of the QED cross sections. The uncertainty in this calculation gives rise to a systematic theoretical error in luminosity common to all experiments, shown in table 1.

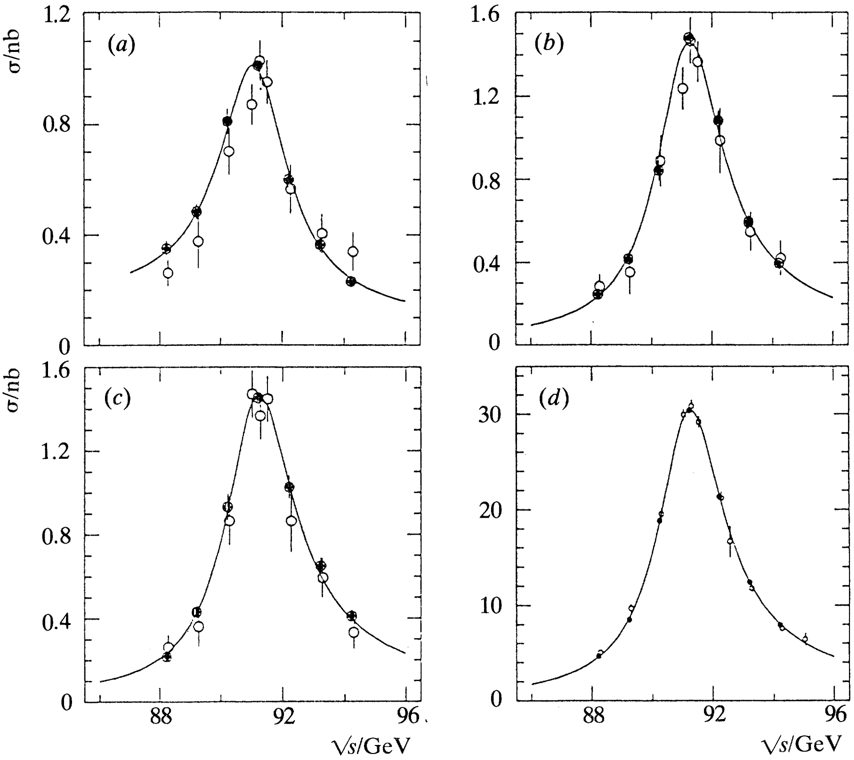

Figure 1. Measurements by OPAL of the cross sections as a function of centre-of-mass energy for (*a*) $e^+e^- \rightarrow e^+e^-$ ($-0.7 < \cos\theta < 0.7$), (*b*) $e^+e^- \rightarrow \mu^+\mu^-$, (*c*) $e^+e^- \rightarrow \tau^+\tau^-$ and (*d*) $e^+e^- \rightarrow$ hadrons. Open circles show 1989 data, full circles 1990 data. The curves are the result of a combined fit assuming lepton universality.

Table 2. *Leptonic event selection; polar angular range included, numbers of events and percentage systematic error*

channel	experiment	$\cos\theta$ range	number of events	systematic error (%)
e^+e^-	ALEPH	-0.9, $+0.7$	—	0.4
	DELPHI	± 0.69	2615	0.9
	L3	± 0.72	4715	0.6
	OPAL	± 0.70	5415	0.7
$\mu^+\mu^-$	ALEPH	± 0.90	—	0.6
	DELPHI	± 0.73	2489	0.8
	L3	± 0.80	3142	0.8
	OPAL	± 0.95	7240	0.5
$\tau^+\tau^-$	ALEPH	± 0.90	—	0.8
	DELPHI	± 0.73	2039	1.4
	L3	± 0.70	2540	2.0
	OPAL	± 0.90	5559	1.2
l^+l^-	ALEPH	± 0.90	—	0.3
	DELPHI	± 0.69	9676	0.7

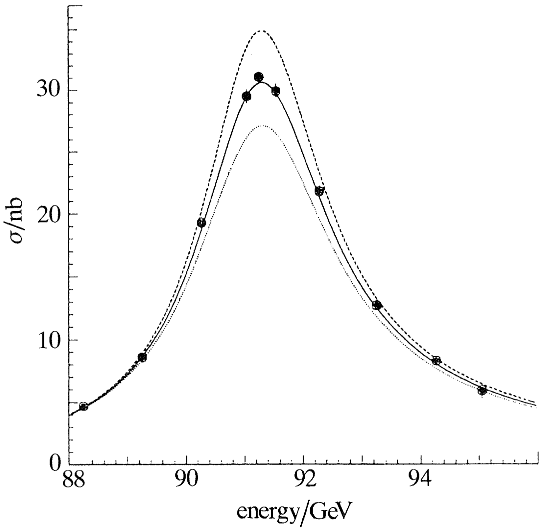

Figure 2. Measurement by ALEPH of the cross section as a function of centre-of-mass energy for $e^+e^- \rightarrow$ hadrons. The Standard Model predictions for $N_\nu = 2$, 3 and 4 are shown. – – –, $N_\nu = 2$; ——, $N_\nu = 3$; · · · · ·, $N_\nu = 4$.

(b) *The* e^+e^- *centre-of-mass energy*

Data corresponding to about half the total integrated luminosity have been collected with LEP running at an energy close of the mass of the Z^0. The remaining data come from a scan taken with e^+e^- centre-of-mass energies at ±1 GeV, ±2 GeV and ±3 GeV from the central value. The centre-of-mass energy is determined by the LEP accelerator group using two different techniques. In the first, field measurements are made on a reference magnet connected in series with the main ring magnets during each LEP fill, and the main ring bending magnets (subject to ageing because of their steel-concrete cores) are tracked relative to the reference magnet by flux loops embedded in their poles. Corrections have to be made to take into account, for example, the orbit of the LEP beam during data-taking, a thin layer of magnetized nickel in the LEP beampipe and the Earth's magnetic field (Hatton *et al.* 1990).

The second absolute energy calibration comes from injecting protons and positrons into a central LEP orbit at 20 GeV and measuring their relative velocities. From this, a correction factor appropriate for 45 GeV e^+e^- beams is extracted (Hatton *et al.* 1990).

The uncertainty in the absolute value of the centre-of-mass energy is a systematic error common to all experiments and is at present estimated as ±0.02 GeV. The most significant contributions come from the correction for the nickel layer in the beampipe and the scaling of results from 20 to 45 GeV.

(c) *Results*

Each experiment has produced hadronic and leptonic lineshape results. The quality of the data is illustrated in figures 1 and 2.

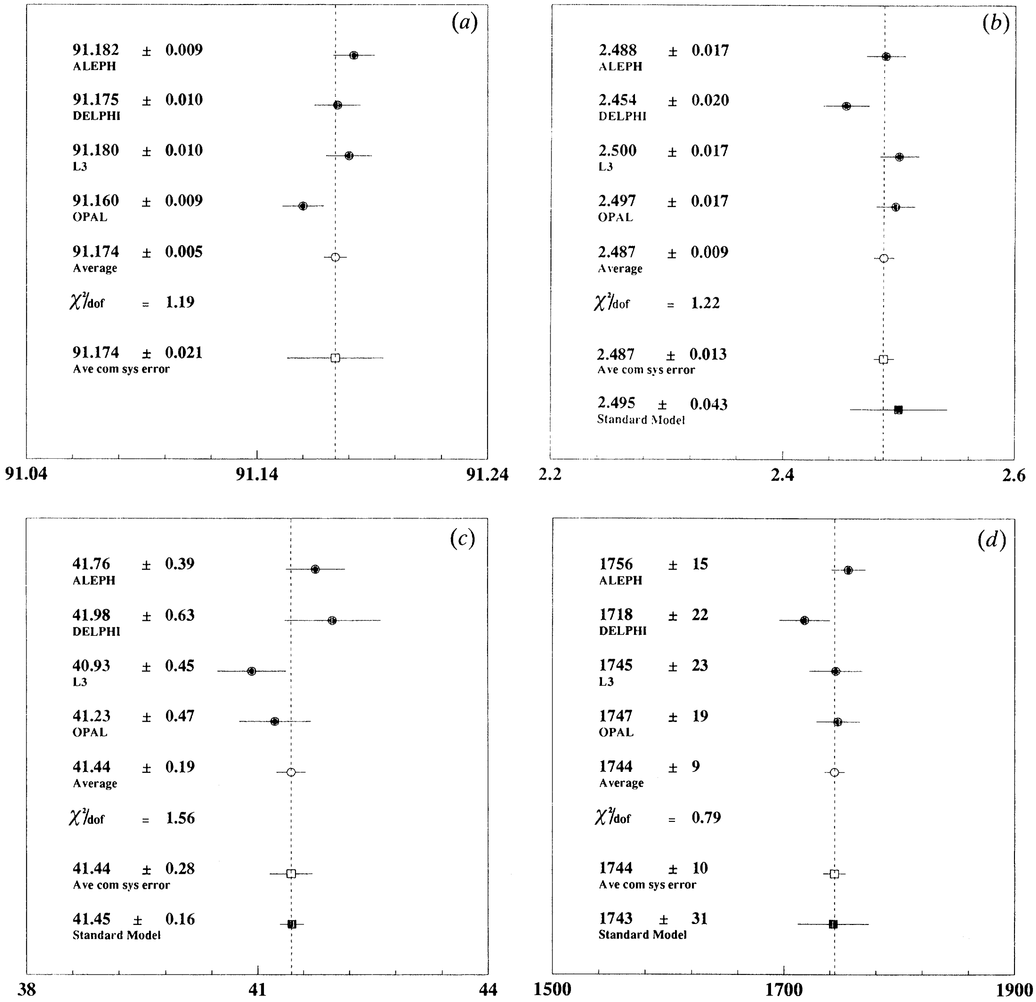

Figure 3. Results for (*a*) the Z^0 mass, M_Z (GeV), (*b*) the total width, Γ_Z (GeV), (*c*) the peak hadronic cross section after correction for initial state radiation, $\sigma^{\mathrm{pole}}_{\mathrm{had}}$ (nb), (*d*) the hadronic width Γ_{had} (MeV). Common systematic errors are in (*a*) 0.02 MeV for the overall LEP energy scale, in (*b*) 0.01 MeV for uncertainties in radiative corrections and the point-to-point energy scale, in (*c*), (*d*) theoretical uncertainties in the luminosity calculation.

3. The decay widths and mass of the Z^0

(*a*) *Theoretical formulation*

The predicted resonant cross section for the production of a fermion–antifermion pair ($e^+e^- \rightarrow f\bar{f}$) can be described to a good approximation by the model-independent formula (Berends *et al.* 1989):

$$\sigma_{f\bar{f}}(s) = \frac{S}{(S-M_Z^2)^2+S^2\Gamma_Z^2/M_Z^2}\left[\frac{12\pi\Gamma_{ee}\Gamma_{f\bar{f}}}{M_Z^2}+I_f\frac{N_c(S-M_Z^2)}{S}\right]+\frac{4N_c\pi Q_f^2\alpha^2(S)}{3S}, \qquad (2.1)$$

where Γ_{ee}, $\Gamma_{f\bar{f}}$ are the partial widths for $Z^0 \rightarrow e^+e^-$, $Z^0 \rightarrow f\bar{f}$, Q_f is the electric charge of the fermion and N_c its colour factor (1 for leptons, 3 for quarks). S is the square of the centre-of-mass energy, M_Z, Γ_Z are the mass and total width of the Z^0 and α is the effective QED coupling constant.

The three terms making up (2.1) describe the contributions of Z^0 exchange, represented by a Breit–Wigner function, of photon exchange and of their interference, with coefficient I_f. The interference term is small.

The effects of initial state radiation are well known but large (*ca.* 30% at the peak), and must be included before comparison can be made with the measured cross section. This is achieved by convoluting expression (2.1) with an appropriate radiator function (Berends *et al.* 1989). In the case of the $e^+e^- \rightarrow e^+e^-$ cross section there are further contributions from t-channel Feynman diagrams and from s- and t-channel interference. All experiments calculate and allow for these additional contributions using the program ALIBABA (Beenakker *et al.* 1990) with an uncertainty which is taken to be 0.5%. Finally, for hadronic final states there is a QCD correction of *ca.* 4%, dependent upon the value of the strong coupling constant α_s.

(*b*) *Results*

The fitting techniques used by the different experiments vary in detail, but all use their lineshape data to determine the mass of the Z^0, M_Z, its total width, Γ_Z, and the peak hadronic cross section, $\sigma^{\text{pole}}_{\text{had}}$, due to Z^0 exchange when unfolded from initial state radiation:

$$\sigma^{\text{pole}}_{\text{had}} = (12\pi/M_Z^2)(\Gamma_{ee}\Gamma_{\text{had}}/\Gamma_Z^2). \tag{3.1}$$

The Z^0 partial decay widths into hadrons, Γ_{had}, muons, $\Gamma_{\mu\mu}$, taus, $\Gamma_{\tau\tau}$ and electrons, Γ_{ee} are also fitted, and the decay width into leptons, $\Gamma_{l^+l^-}$ is obtained with the further assumption of lepton universality.

The quality of some fits to the data are indicated in figure 1, and the results for each experiment are displayed in figures 3, 4 and 5, together with combined results for all LEP experiments. For the latter, any common systematic error coming from the theoretical uncertainty in the calculation of luminosity is removed before a weighted mean is taken of the results of the individual experiments. The chi-squared per degree of freedom of the data about the mean is shown. The common error is then added back in quadrature to the error on the weighted mean.

Also shown in figures 3–5 are the predictions of the Standard Model using as input $M_Z = 91.174$ GeV/c^2, $\alpha_s = 0.12$ and values for the unknown mass of the Higgs boson, M_H and the top quark, M_t, of $M_H = 100$ GeV/c^2, $M_t = 150$ GeV/c^2. The error on the prediction is to be interpreted as the change caused by varying these quantities within the ranges $0.11 < \alpha_s < 0.13$, $50 < M_t < 250$ GeV/c^2, $50 < M_H < 1000$ GeV/c^2. The Standard Model calculations are made with the program ZFITTER (Bardin *et al.* 1989). The individual Z^0 widths have a fairly strong dependence theoretically on M_t which, however, largely cancels in their ratio. The quantities $\sigma^{\text{pole}}_{\text{had}}$ and $R_Z = \Gamma_{\text{had}}/\Gamma_{l^+l^-}$ therefore provide good tests of the Standard Model.

It is seen that for all measurements the experiments agree well among themselves, with lepton universality and with the predictions of the Standard Model, the latter comparison being summarized in figure 5*d*. The importance of achieving further reductions in systematic errors is evident as these now dominate some measurements.

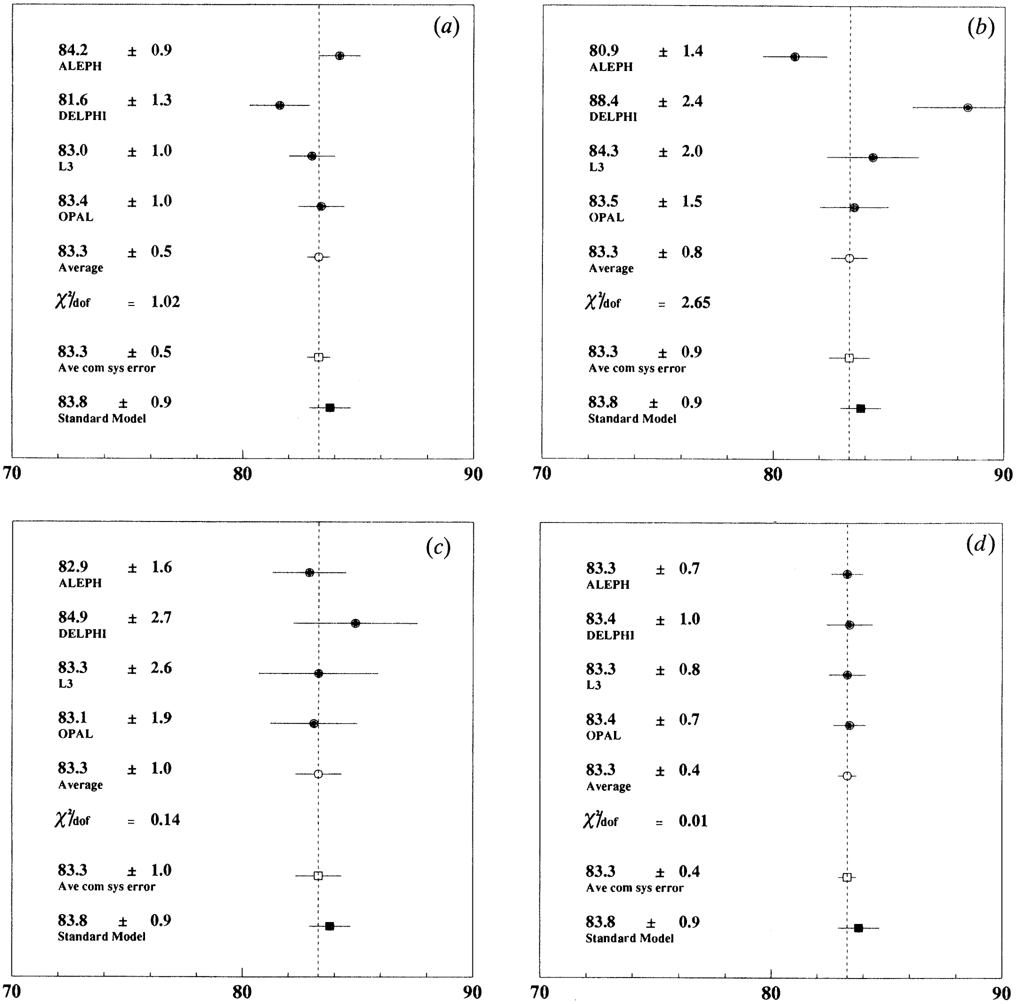

Figure 4. Results for the Z^0 partial decay widths (MeV) into (*a*) e^+e^-, (*b*) $\mu^+\mu^-$, (*c*) $\tau^+\tau^-$, (*d*) l^+l^-, assuming lepton universality.

4. The number of light neutrino generations

(*a*) *Determination from the lineshape measurements*

(i) Under the assumption of lepton universality, the Z^0 invisible width, Γ_{inv}, is deduced from

$$\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_{l^+l^-}. \tag{4.1}$$

The values found by each experiment are displayed in figure 5*b*.

The number of light neutrino species, N_ν, can be obtained from $\Gamma_{\text{inv}} = N_\nu \Gamma_{\nu\bar{\nu}}$, where $\Gamma_{\nu\bar{\nu}}$ is the partial decay width into each generation of light neutrino. Taking the value $\Gamma_{\nu\bar{\nu}} = 166.9^{+2.2}_{-1.4}$ MeV from the Standard Model (the error coming from the uncertainty in the top quark mass), and the LEP experimental value $\Gamma_{\text{inv}} = 493 \pm 13$ MeV gives

$$N_\nu = 2.95 \pm 0.08\,(\text{exp})^{+0.02}_{-0.04}\,(M_t).$$

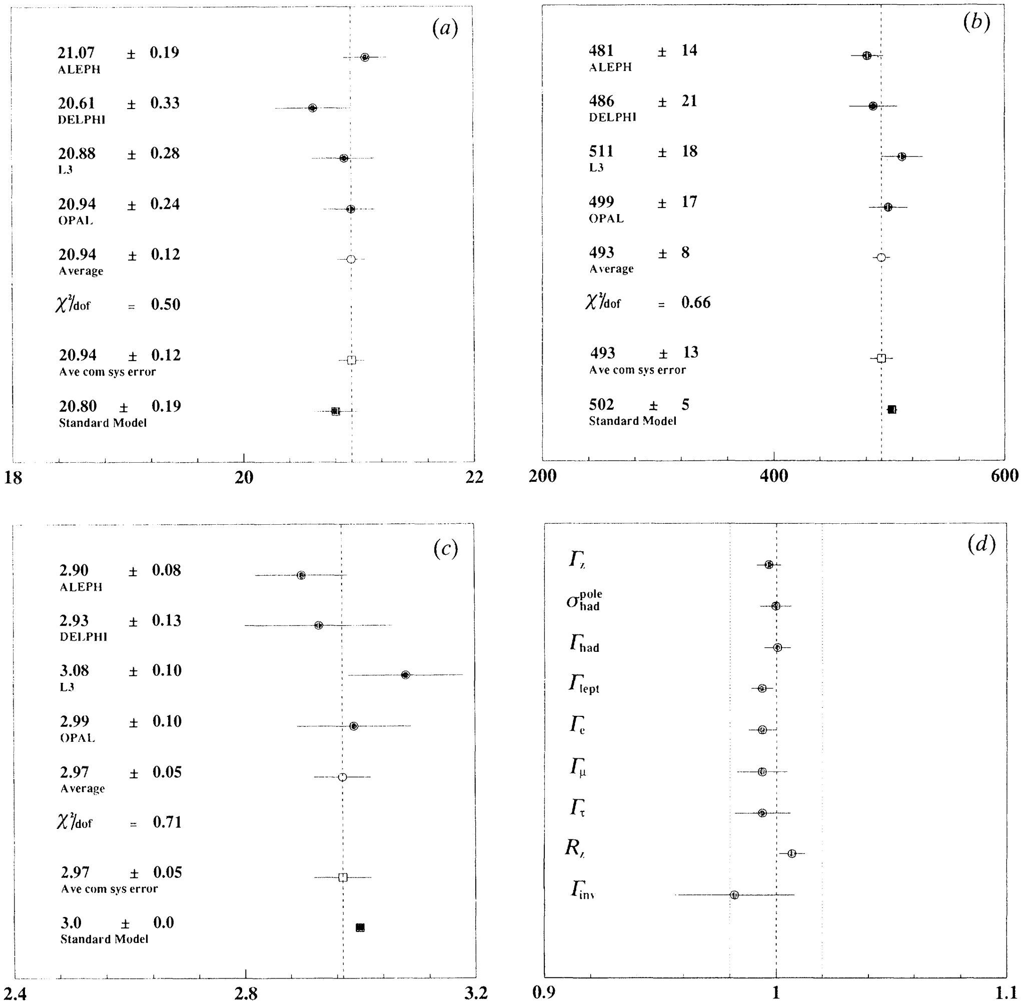

Figure 5. Results for (*a*) the ratio of hadronic to leptonic partial widths, R_Z, (*b*) the Z^0 partial decay width (MeV) into invisible final states, (*c*) the number of light neutrino species, (*d*) the ratio of the average LEP result to the prediction of the Standard Model for each of the quantities shown in figures 3, 4 and 5. The errors in (*d*) exclude uncertainties in the Standard Model predictions due to M_t, M_H and α_s.

(ii) An alternative approach is to use expression (3.1) for the Z^0 resonance peak cross section together with (4.1) to give

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu\bar{\nu}}} = \frac{\Gamma_{l^+l^-}}{\Gamma_{\nu\bar{\nu}}}\left[\sqrt{\left(\frac{12\pi R_Z}{M_Z^2\,\sigma^{\text{pole}}_{\text{had}}}\right)} - R_Z - 3\right].$$

The ratio $\Gamma_{l^+l^-}/\Gamma_{\nu\bar{\nu}}$ is taken from the Standard Model, with the advantage that it is almost independent of M_t, as well as still being valid if any unexpected states yielding hadrons are present in Z^0 decay. The combined result of the experiments is

$$N_\nu = 2.97 \pm 0.05.$$

Thus the lineshape data support the existence of just three generations of light neutrino, with mass less than half that of the Z^0, in accord again with the minimal Standard Model. This is further illustrated by the lineshape predictions shown in figure 2.

(*b*) *Direct measurement of* $\Gamma_{\nu\bar{\nu}}$

The OPAL Collaboration (Akrawy *et al.* 1990*b*) has recently reported the results of a direct determination of the Z^0 invisible width through measuring the cross section for the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$, with a single photon produced from initial state radiation. This is a complementary method of measuring $\Gamma_{\nu\bar{\nu}}$, independent of the assumption that all visible decays of the Z^0 are accounted for in the experimental analysis of charged leptons and hadronic events. The direct measurement is easier at beam centre-of-mass energies considerably above the Z^0 peak, when initial state photon radiation brings the energy back towards the Z^0 resonance and so is strongly favoured. The photons are also then of high energy and more readily detected. Nevertheless, the data coming from the existing LEP energy scan yield 73 candidate events with photon energy of more than 1.5 GeV and the results:

$$\Gamma_{\nu\bar{\nu}} = 0.50 \pm 0.07 \pm 0.03 \text{ GeV},$$

$$N_\nu = 3.0 \pm 0.4 \pm 0.2,$$

where the first error is statistical and the second systematic. The present measurement is clearly statistics limited, but is in full agreement with the invisible width determined from the Z^0 lineshape data.

5. Summary

Precise measurements of the mass and widths of the Z^0 have resulted from the first year of data-taking at LEP, and the number of light neutrino species into which the Z^0 decays has been determined to be three. Results are in excellent agreement between the four experiments and with the predictions of the Standard Model. The latter have uncertainties due to lack of knowledge of the mass of the top quark and the Higgs boson and the present data, along with measurements of angular distributions of leptonic final states, can be used to provide limits on the top quark mass. This topic is covered elsewhere in this discussion meeting.

I am grateful to physicists from all four LEP experimental collaborations for providing unpublished data for use in this review.

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